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# Quantum Computing

By Scott Pakin

A *bit* can be

**true** or **false**

**left** or **right**

**up** or **down**

**1** or **0**

**set** or **reset**

**on** or **off**

**high** or **low**

It is the most primitive unit of information

**yes** or **no**

**in** or **out**

# $N$ bits can represent any one of $2^N$ values

➡ 000

001

010

011

➡ 100

101

110

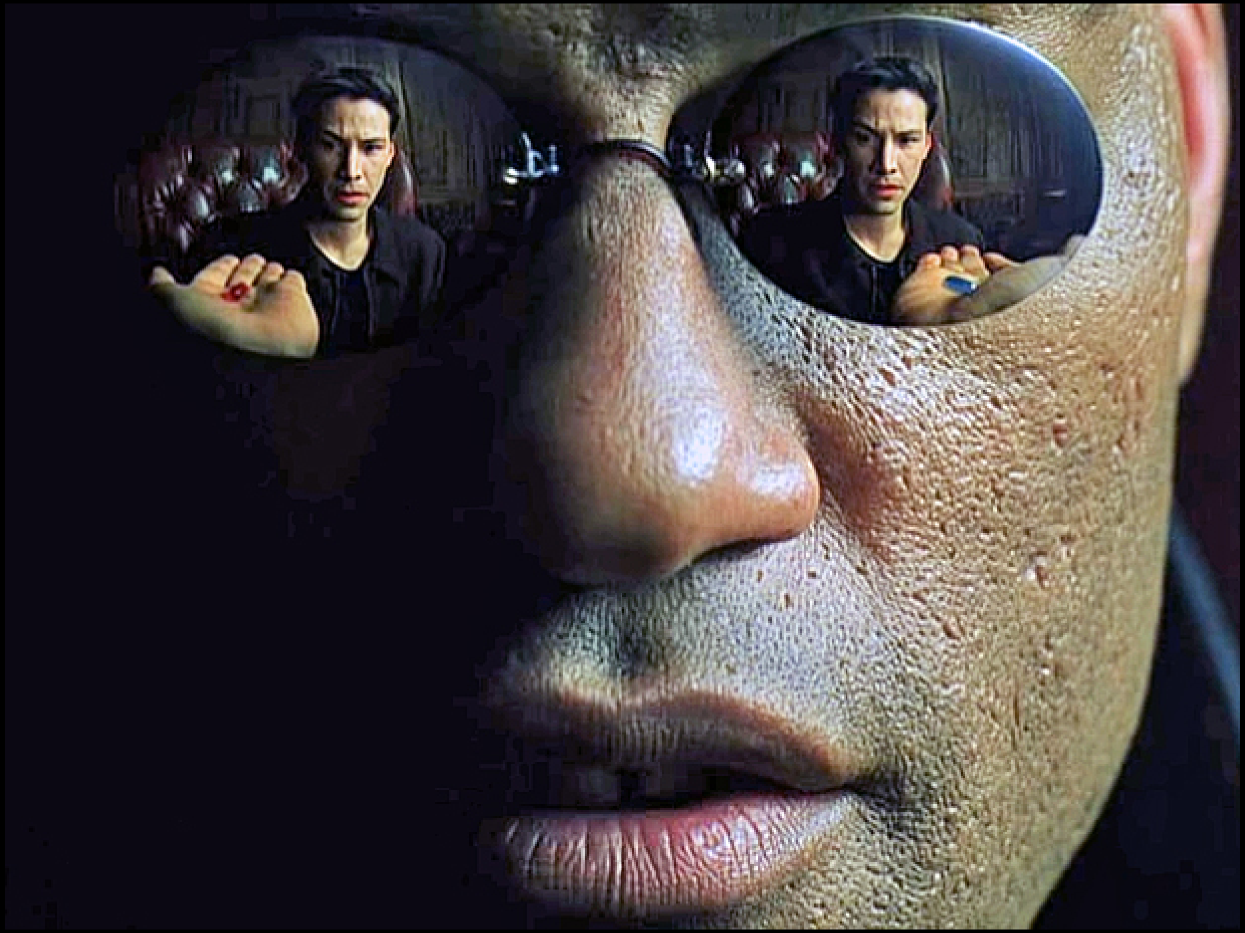
111

There are **four** possible 1-bit operators

- $a$
- $\bot$
- $\top$
- $\neg a$

# There are **sixteen** possible 2-bit operators

- ⊥
- a↓b
- a↚b
- ¬a
- a↛b
- ¬b
- a⊻b
- a↑b
- a∧b
- a↔b
- b
- a→b
- a
- a←b
- a∨b
- T



You take the blue pill, the story ends; you wake up in your bed and believe whatever you want to believe. You take the red pill, you stay in Wonderland, and I show you how deep the rabbit hole goes.

# A *qubit* is a point in a 2-D Hilbert space

(i.e., a pair of complex numbers)

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \begin{pmatrix} i - \frac{3}{2} \\ -\frac{\sqrt{12i - 1}}{2} \end{pmatrix}$$

# A qubit's state as a linear combination of basis vectors:

How much "0-ness"

How much "1-ness"

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

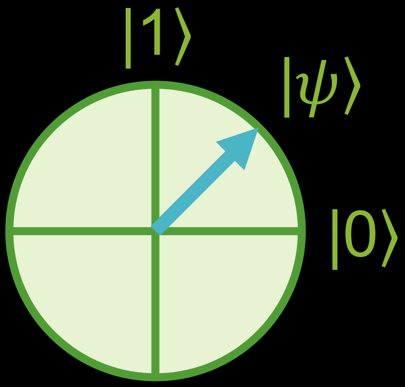
$$|0\rangle \qquad |1\rangle$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\alpha, \beta \in \mathbb{C}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

# A qubit can simultaneously have properties of both 0 and 1

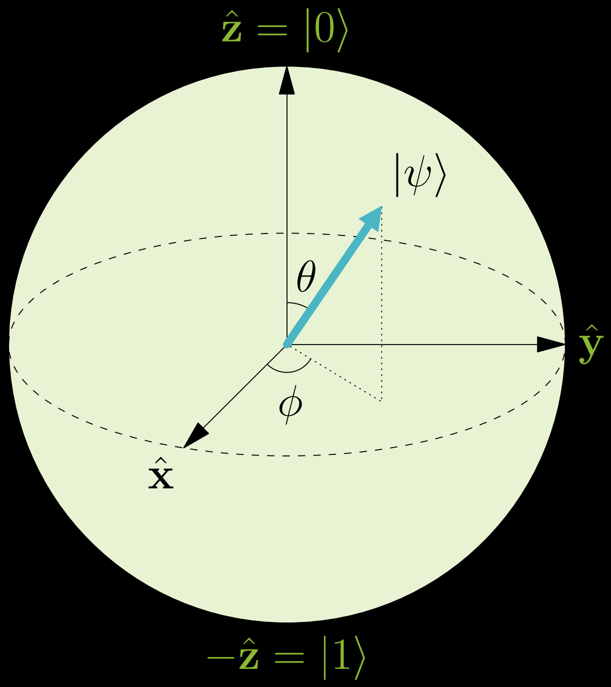


We call $|\psi\rangle$ a *superposition* of 0 and 1

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

# There are an infinite number of 0s and 1s

Easier to discern from the *Bloch sphere*, a commonly used projective vector space



All $e^{i\phi}|0\rangle$ represent different *phases* of 0

$N$ qubits can represent **all** $2^N$ values simultaneously

- ➡ 000
- ➡ 001
- ➡ 010
- ➡ 011
- ➡ 100
- ➡ 101
- ➡ 110
- ➡ 111

$N$ qubits are represented with a vector of length $2^N$

# Measuring a qubit collapses it to a classical 0 or 1

$\alpha|0\rangle + \beta|1\rangle$ is measured as 0 with probability $|\alpha|^2$ and 1 with probability $|\beta|^2$

**Oracle**: I'd ask you to sit down, but, you're not going to anyway. And don't worry about the vase.
**Neo**: What vase?

[*Crash*]

**Oracle**: That vase.

…

**Neo**: How did you know?
**Oracle**: Ohh, what's really going to bake your noodle later on is, would you still have broken it if I hadn't said anything?

# A 2-qubit state can be constructed from the tensor product of two 1-qubit states

(and similarly for $N$-qubit states)

$$|10\rangle = |1\rangle \otimes |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \cdot 1 \\ 0 \cdot 0 \\ 1 \cdot 1 \\ 1 \cdot 0 \end{pmatrix} = \begin{array}{c} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \end{array} \begin{array}{l} \text{00-ness} \\ \text{01-ness} \\ \text{10-ness} \\ \text{11-ness} \end{array}$$

# Qubits do not necessarily have their own identity

$\begin{pmatrix}\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{pmatrix}^{\mathbf{T}}$ is read as 00, 01, 10, or 11 with 25% probability apiece

$\begin{pmatrix}\mathbf{0} & \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} & \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} & \mathbf{0}\end{pmatrix}^{\mathbf{T}}$ is read as 01 or 10 with 50% probability apiece

Measuring/modifying one qubit affects the other

We call this *entanglement*

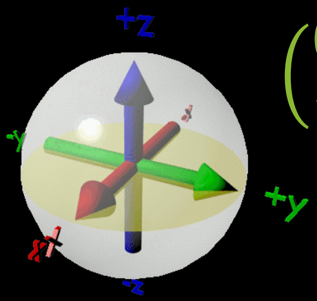
# There are **infinitely many** 1-qubit operators

(2×2 unitary matrices)

reversible

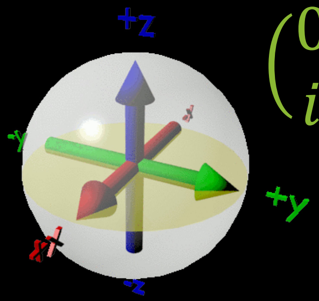
- Example #1: $NOT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- Example #2: $\sqrt{NOT} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$

# Visualizing some 1-qubit operators (“gates”)



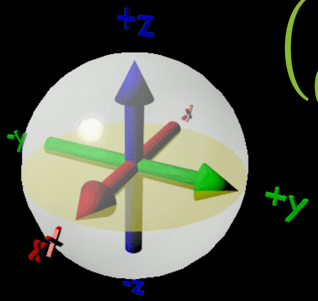
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Pauli *x* gate (*X*)



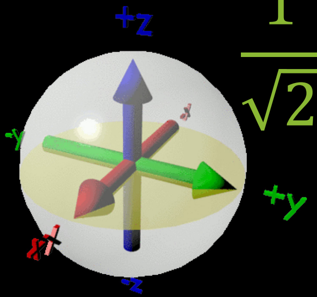
$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Pauli *y* gate (*Y*)



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli *z* gate (*Z*)



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hadamard gate (*H*)

- A Hadamard gate puts a qubit in a perfect superposition of 0 and 1
- $XX = YY = ZZ = HH = I$
- Implication: deterministic → random → deterministic

# There are **infinitely many** 2-qubit operators

(4×4 unitary matrices)

- Example #2:

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

- Example #3:

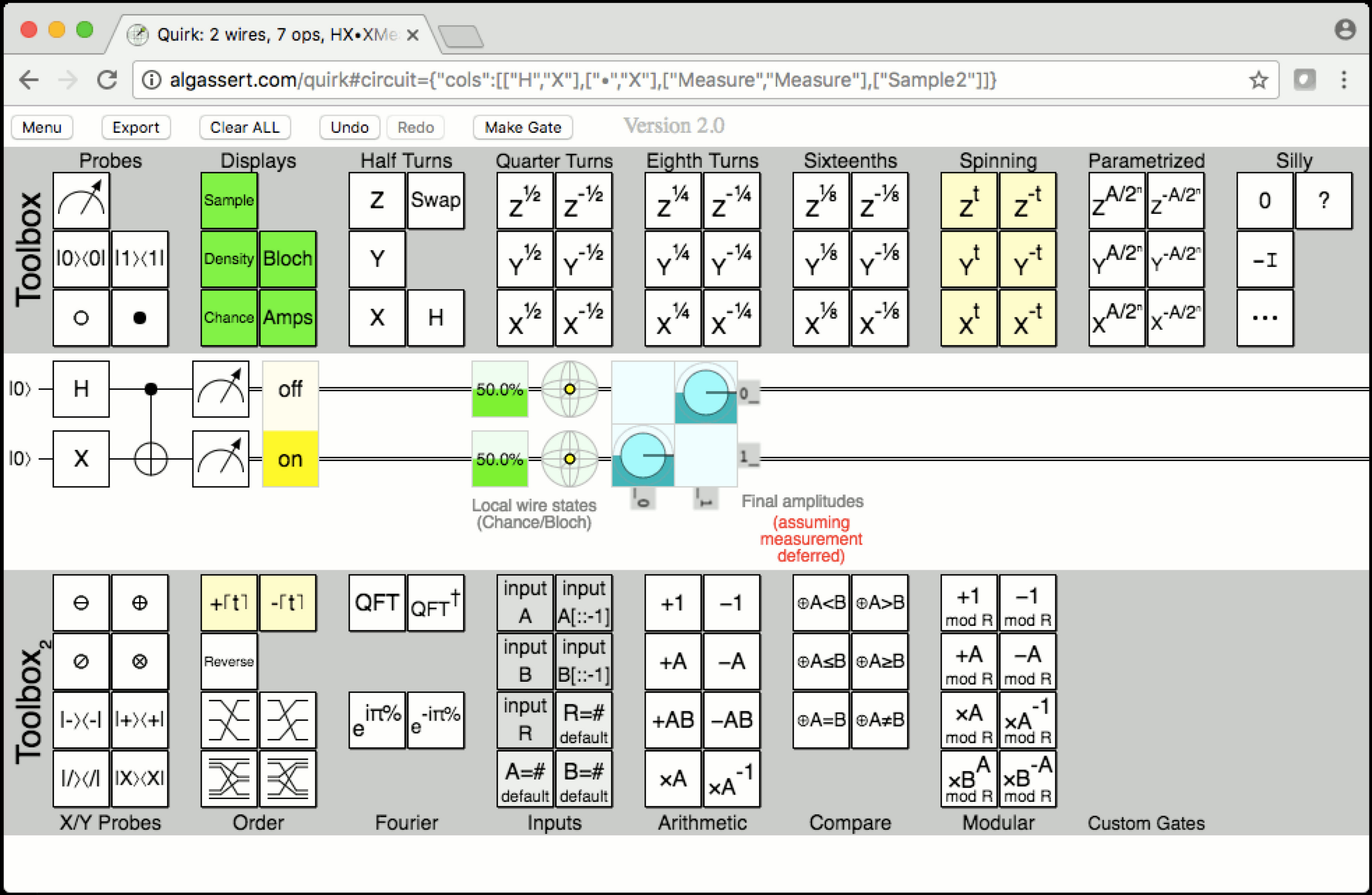
$$SWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$iSWAP = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Negate b if a = |1⟩
- Swaps the values of qubits a and b
- Useful for entangling
- Can construct [illegible] gates
- What if b is in a superposition?
- What if a is in a superposition?

I know what you’re thinking, ’cause right now I’m thinking the same thing.  Actually, I’ve been thinking it ever since I got here: Why, oh why, didn’t I take the **blue** pill?

# Quantum circuits

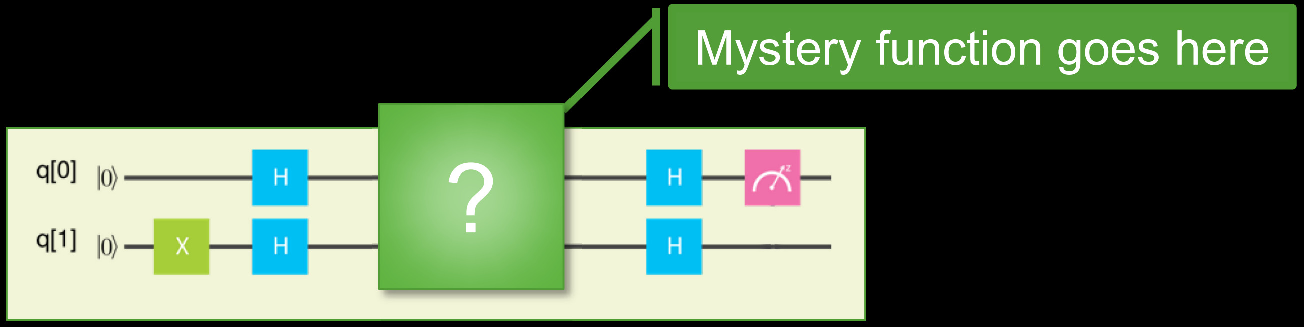


Quirk: 2 wires, 7 ops, HX•XMe

algassert.com/quirk#circuit={"cols":[["H","X"],["•","X"],["Measure","Measure"],["Sample2"]]}

# What’s the big deal?

- First answered by Deutsch and Josza in 1992



Mystery function goes here

- Determine if a given black-box function is constant or balanced
  - For one bit, constant functions are $f(x)=0$ and $f(x)=1$; balanced are $f(x)=x$ and $f(x)=\neg x$
  - *Classical*: Evaluate $f(x)$ twice
  - *Quantum*: Evaluate $f(x)$ once—returns 0 for balanced, 1 for constant
- Increasing performance improvement with scale
  - *Classical*: Evaluate $f(x)$ $\lfloor N/2+1 \rfloor$ times for $N$ bits
  - *Quantum*: Evaluate $f(x)$ once for $N$ bits

# Quantum algorithms

- Begin and end classically (i.e., only $|0\rangle$ and $|1\rangle$ states)
- Quantum in between
- Can compute on all $2^N$ combinations in parallel
- The catch: Only one $N$-bit answer comes out

# Challenges

- Reduce/cancel out probability amplitudes of non-solutions
- Manage rotations in an $N$-dimensional Hilbert space
- To date, only a small number of algorithms exist

| **Speedup over classical** | **#** |
|---|---|
| Exponential | 2 |
| Superpolynomial | 27 |
| Polynomial | 25 |
| Constant | 2 |
| Varies | 4 |
| *Total* | *60* |

Stephen Jordan
*Quantum Algorithm Zoo*
http://math.nist.gov/quantum/zoo

# Unordered search

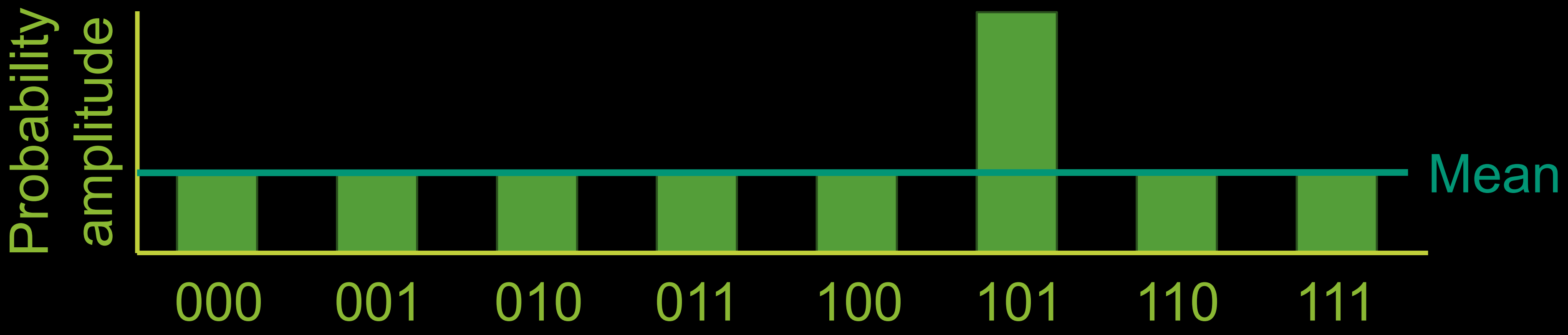
- Which screen’s image matches a given pattern?
- Classical: $O(N)$ queries
- Quantum: $O(\sqrt{N})$ queries (next two slides)

# Grover's search algorithm

- Given
  - A power-of-2 number of elements
  - A guarantee that exactly one element matches the pattern
  - An operator $U_\omega$ that, given an element $|x\rangle$, flips the probability amplitude iff the element matches (i.e., $U_\omega|x\rangle = -|x\rangle$ for $x = \omega$ and $U_\omega|x\rangle = |x\rangle$ for $x \neq \omega$)
- Return the matching element

# Grover's search algorithm

- **Approach:** For $\sqrt{N}$ iterations, alternately apply $U_\omega$ followed by "Grover diffusion operator" $U_s$
- $U_s \equiv 2|s\rangle\langle s| - I$, which flips amplitudes around the mean

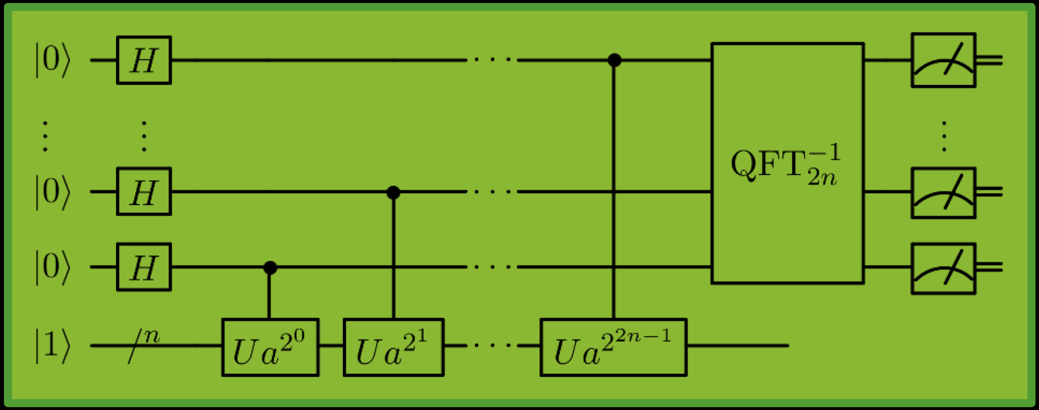


# Integer factorization

- Factor an integer into a product of two primes
- Best known classical algorithm has running time $O(2^{\sqrt[3]{N}})$
- Best known quantum algorithm has running time $O(\log^3 N)$ (next slide)
- Exponential speedup
- Expected that factoring a 50-100 bit number would be intractable classically but tractable with quantum: “quantum advantage”

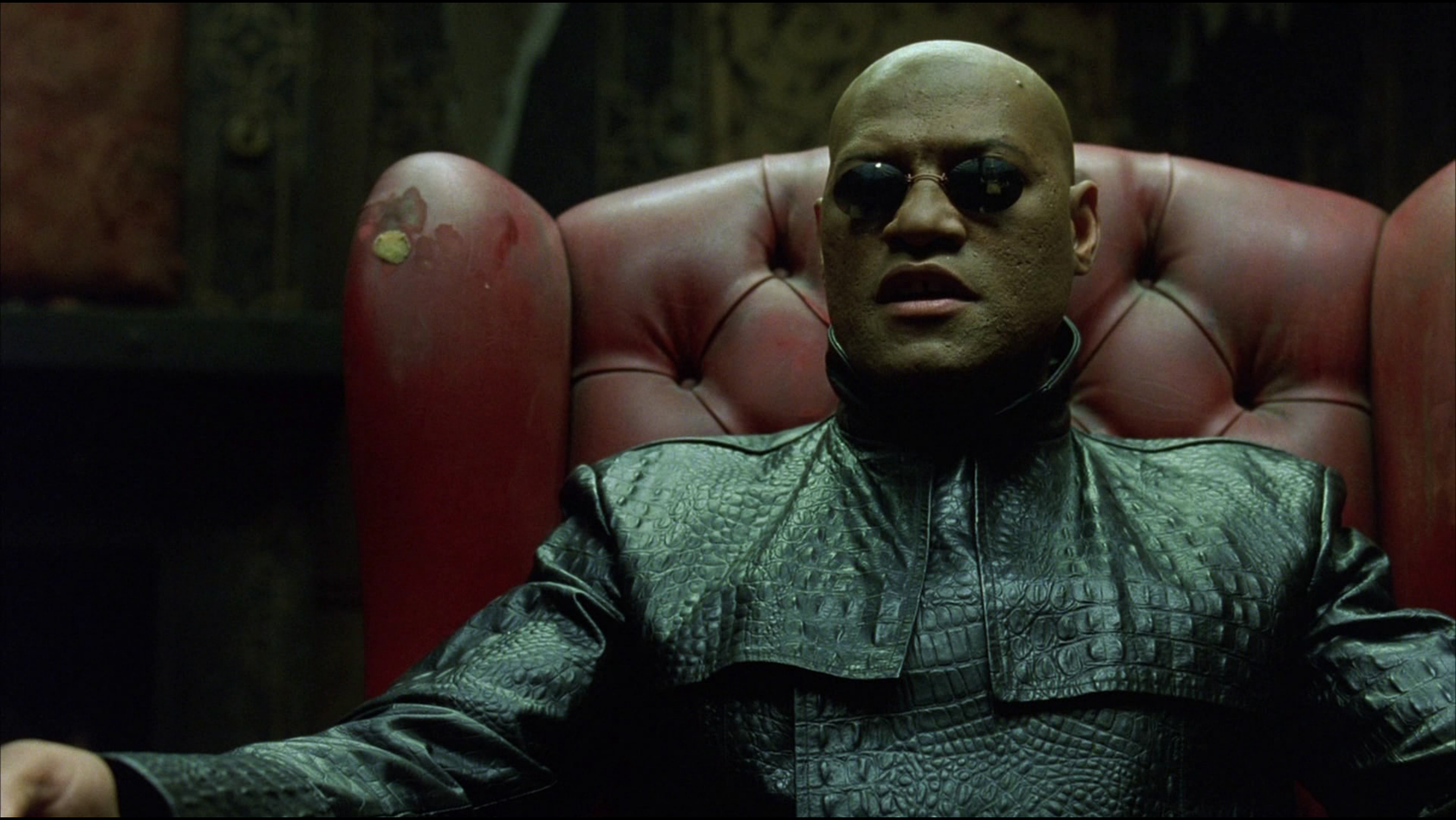
# Shor's algorithm

- It's not too hard to factor $N$ into primes $p$ and $q$ if we know the period of the sequence {$a^1$ mod $N$, $a^2$ mod $N$, $a^3$ mod $N$, …} for some $a<N$ with $p$ł$a$ and $q$ł$a$
- Apply an inverse quantum Fourier transform to find the period
- All else is classical—and randomized



# Conclusions

- *Very* different form of computing
- Qubits carry more information than classical bits (e.g., phase)
- Quantum gates perform state transformations in high-D spaces
- Exploit superpositioning and entanglement for full parallelism
- Manipulate probability amplitudes to isolate correct answers
- Potential for *exponential* performance improvement



I’m trying to free your mind, Neo.  But I can only show you the door.  You’re the one that has to walk through it.